$$[T]_{\mathcal{B}'_{can} \leftarrow \mathcal{B}_{can}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

$$T(1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad T(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad T(t^2) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

THM (8.10)

Pour

$$[T(9-2t+7t^2)]_{\mathcal{B}'_{can}} = [T]_{\mathcal{B}'_{can} \leftarrow \mathcal{B}_{can}} \cdot [9-2t+7t^2]_{\mathcal{B}_{can}}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

Déf. 8.15 Soient $\mathcal{B}$ et $\mathcal{C}$ deux bases d'un EV V. Alors la matrice de passage de $\mathcal{B}$ vers $\mathcal{C}$ est $[id]_{\mathcal{C}\leftarrow\mathcal{B}}$ où $id: V \to V$ est l'application identité (i.e. $id(v) = v, \forall v \in V$)

Si $\mathcal{B} = \{v_1, \dots, v_n\}$ et $\mathcal{C} = \{w_1, \dots, w_n\}$, alors

$$[id]_{\mathcal{C}\leftarrow\mathcal{B}} = \left[[v_1]_{\mathcal{C}} \cdots [v_n]_{\mathcal{C}} \right]$$

Notation On va noter $P_{\mathcal{C}\leftarrow\mathcal{B}}$ cette matrice.

Prop. 8.16 (Coro de THM 8.10) :

$$[v]_{\mathcal{C}} = P_{\mathcal{C}\leftarrow\mathcal{B}} \cdot [v]_{\mathcal{B}} \quad \text{pour tout } v \in V$$

base $\mathcal{B}$ et $\mathcal{B}$ de V

EXM 8.14

$$\mathcal{B} = \left\{ \overbrace{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}^{\bar{v}_1}, \overbrace{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}^{\bar{v}_2} \right\}$$

$$\mathcal{B}_{can} = \left\{ \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\bar{e}_1}, \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\bar{e}_2} \right\}$$

Calculer $P_{\mathcal{B} \leftarrow \mathcal{B}_{can}}$ et $P_{\mathcal{B}_{can} \leftarrow \mathcal{B}}$

$$\underbrace{P_{\mathcal{B}_{can} \leftarrow \mathcal{B}}}_{} = \begin{bmatrix} [\bar{v}_1]_{\mathcal{B}_{can}} & [\bar{v}_2]_{\mathcal{B}_{can}} \end{bmatrix}$$

$\forall \bar{v} \in \mathbb{R}^n$ $\boxed{[\bar{v}]_{\mathcal{B}_{can}} = \bar{v}\ !}$

$$= \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

PROP 8.16 (suite) $P_{\mathcal{C}\leftarrow\mathcal{B}}$ est inversible et

$$P_{\mathcal{B}\leftarrow\mathcal{C}} = P_{\mathcal{C}\leftarrow\mathcal{B}}^{\;-1}$$

Alors $$P_{\mathcal{B}\leftarrow\mathcal{B}can} = P_{\mathcal{B}can\leftarrow\mathcal{B}}^{\;-1} = \begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}^{-1}$$

$$= \frac{1}{1+2}\begin{pmatrix}1 & -2\\ 1 & 1\end{pmatrix} = \begin{pmatrix}1/3 & -2/3\\ 1/3 & 1/3\end{pmatrix}$$

EXM 8.17* $$\mathcal{B} = \left\{ \overbrace{\begin{pmatrix}1\\ -1\end{pmatrix}}^{\vec{v}_1}, \overbrace{\begin{pmatrix}2\\ 1\end{pmatrix}}^{\vec{v}_2} \right\}$$

$$\mathcal{C} = \left\{ \underbrace{\begin{pmatrix}1\\ 1\end{pmatrix}}_{\vec{w}_1}, \underbrace{\begin{pmatrix}1\\ -2\end{pmatrix}}_{\vec{w}_2} \right\}$$

Calculer $P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{pmatrix} 1/3 & 5/3 \\ 2/3 & 1/3 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$\Updownarrow$

$$\begin{cases} \lambda_1 + \lambda_2 = 1 \\ \lambda_1 - 2\lambda_2 = -1 \end{cases} \xRightarrow{\boxed{L_1 - L_2}} \begin{cases} 3\lambda_2 = 2 \Rightarrow \lambda_2 = 2/3 \\ \lambda_1 = 1 - \lambda_2 = 1 - \frac{2}{3} = 1/3 \end{cases}$$

On peut sinon

$$\boxed{P_{\mathcal{C}\leftarrow\mathcal{B}_{can}} \cdot P_{\mathcal{B}_{can}\leftarrow\mathcal{B}} = P_{\mathcal{C}\leftarrow\mathcal{B}}}$$

$$\mathcal{C} \leftarrow \mathcal{B}_{can} \leftarrow \mathcal{B}$$

car

$$P_{\mathcal{B}_{can} \leftarrow \mathcal{B}} = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \quad \text{et} \quad P_{\mathcal{C} \leftarrow \mathcal{B}_{can}} = P_{\mathcal{B}_{can} \leftarrow \mathcal{C}}^{-1}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}^{-1}$$

$$= -\frac{1}{3} \begin{pmatrix} -2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix}$$

$$P_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 & 5/3 \\ 2/3 & 1/3 \end{pmatrix}$$

Prop 8.16 (suite) | Si $\mathcal{B}, \mathcal{C}$ et $\mathcal{D}$ sont

$\mathcal{B}$ base de V, alors $P_{\mathcal{D}\leftarrow\mathcal{B}} = P_{\mathcal{D}\leftarrow\mathcal{E}}\, P_{\mathcal{E}\leftarrow\mathcal{B}}$

EXM 8.18*

$$\mathcal{B} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \right\}$$

$$\mathcal{E} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Calculer $P_{\mathcal{E}\leftarrow\mathcal{B}} := \underbrace{\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{-1}}_{P_{\mathcal{E}\leftarrow\mathcal{B}_{can}}} \cdot \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix}}_{P_{\mathcal{B}_{can}\leftarrow\mathcal{B}}}$

calculé ? finir ?

$$P_{\mathcal{B}_{can} \leftarrow \mathcal{B}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix} \qquad P_{\mathcal{B}_{can} \leftarrow \mathcal{C}} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$P_{\mathcal{C} \leftarrow \mathcal{B}_{can}} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{-1}$$

$$\Rightarrow P_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{pmatrix} 3/2 & -2 & 0 \\ -3 & 4 & 2 \\ -1/2 & 1 & 0 \end{pmatrix}$$

THM 8.19 Soit $T : V \longrightarrow V'$ AL

$\mathcal{B}, \mathcal{C} \subseteq V$ bases de V $\qquad$ $\mathcal{B}', \mathcal{C}' \subseteq V'$ bases de V'

Abs (*)

$$[T]_{\mathcal{C}' \leftarrow \mathcal{C}} = P_{\mathcal{C}' \leftarrow \mathcal{B}'} [T]_{\mathcal{B}' \leftarrow \mathcal{B}} P_{\mathcal{B} \leftarrow \mathcal{C}}$$

De façon graphique :

$$\begin{tikzcd} \mathcal{C}' & & \mathcal{C} \arrow[ll] \arrow[d, "\text{passage}"] \\ \mathcal{B}' \arrow[u, "\text{passage}"] & & \mathcal{B} \arrow[ll] \end{tikzcd}$$

On appelle (*) la formule de changement de base.

$\boxed{\text{Cas spécial}}$

$T : V \longrightarrow V$

$\mathcal{B}$ et $\mathcal{C}$ deux bases de V

$[T]_{\mathcal{C} \leftarrow \mathcal{C}} = P_{\mathcal{C} \leftarrow \mathcal{B}} [T]_{\mathcal{B} \leftarrow \mathcal{B}} \, P_{\mathcal{B} \leftarrow \mathcal{C}}$

$$= P_{\mathcal{B}\leftarrow\mathcal{E}}^{-1}\,[T]_{\mathcal{B}\leftarrow\mathcal{B}}\cdot P_{\mathcal{B}\leftarrow\mathcal{E}}$$

EXM 8.20

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_2 - 5x_3 \\ x_1 + 3x_2 \end{pmatrix}$$

$$\left(T : \mathbb{R}^3 \to \mathbb{R}^2 \right)$$

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$\mathcal{B}' = \left\{ \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\}$$

Calculer $[T]_{\mathcal{B}'\leftarrow\mathcal{B}}$